Examining Almost Bandlimited Graph Signals

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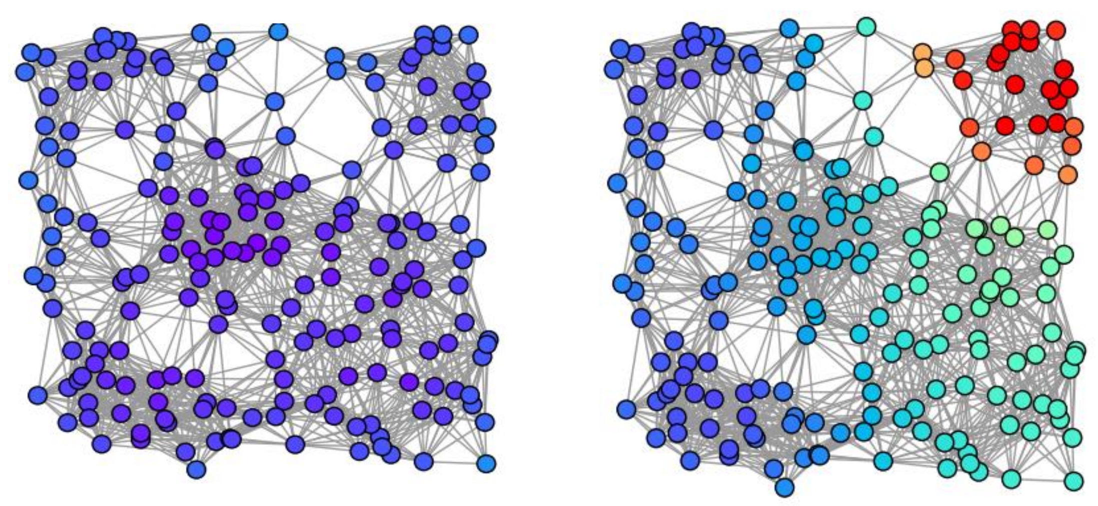
Introduction

Graph signals can be viewed in the frequency domain, using eigenvalues and eigenvectors.



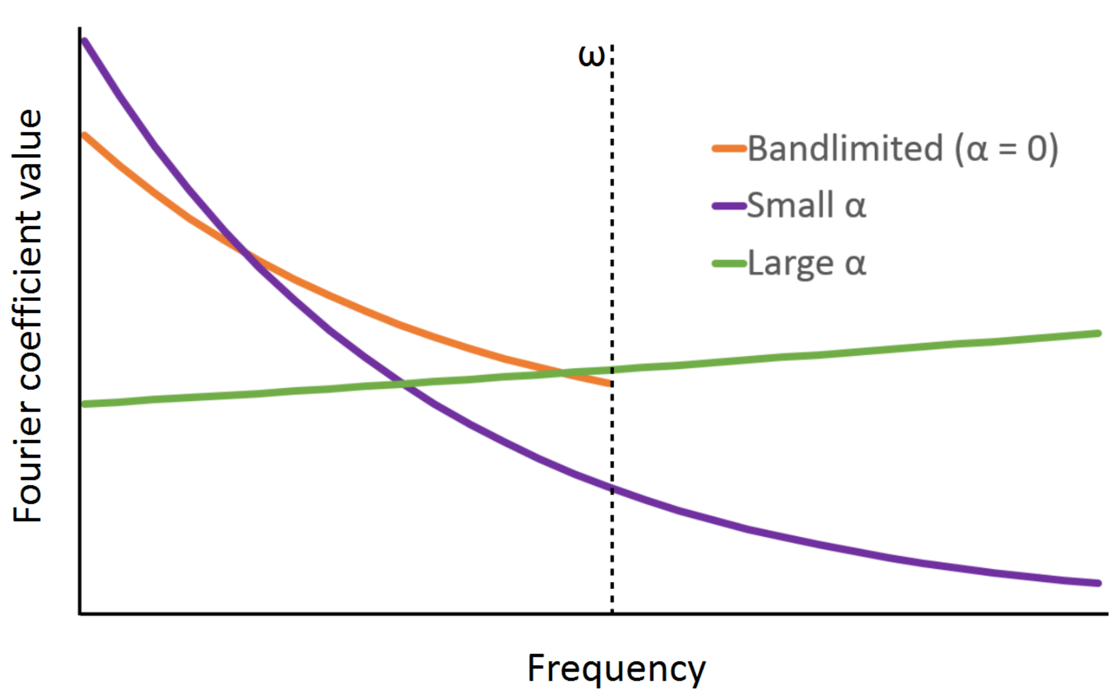
The first and second Laplacian eigenvectors mapped onto a graph.

Optimal sampling and interpolation methods for bandlimited graph signals have been shown in [1,2]. Knowing the amount of high frequency energy in a signal may be helpful for sampling almost bandlimited signals.

Almost Bandlimited Signals

Almost bandlimited signals can be characterized as α-almost ω-bandlimited, or (α,ω)-ABL:

$$\alpha = \frac{\text{energy in frequencies higher than } \omega}{\text{total energy}}$$



The next 3 panels show the results of this research project.

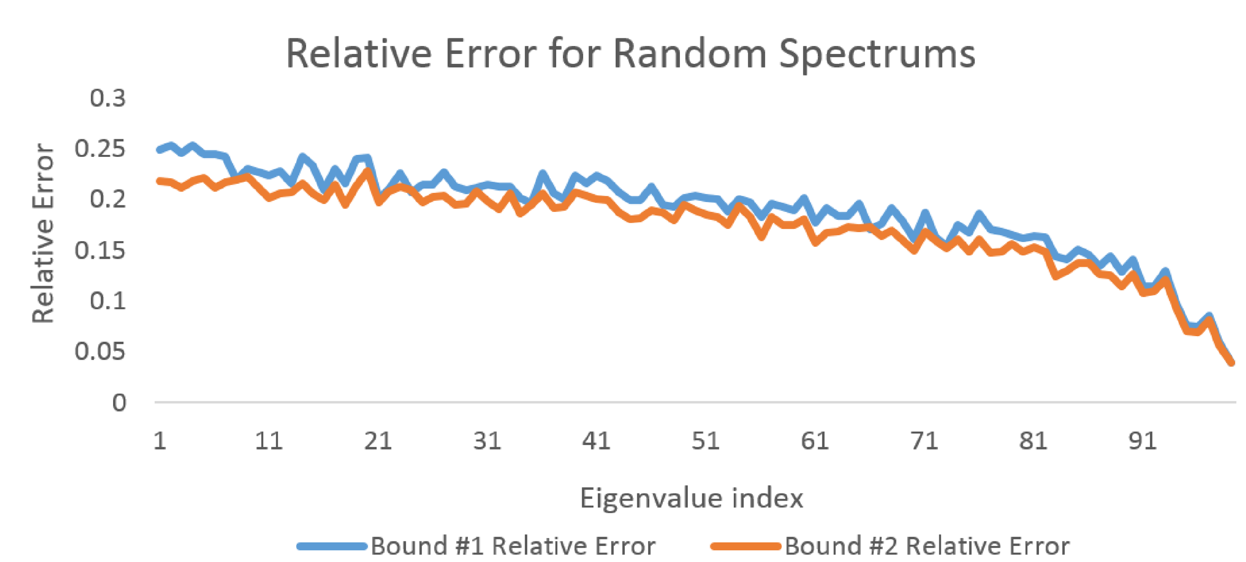
Upperbound on Diffusion Timesteps

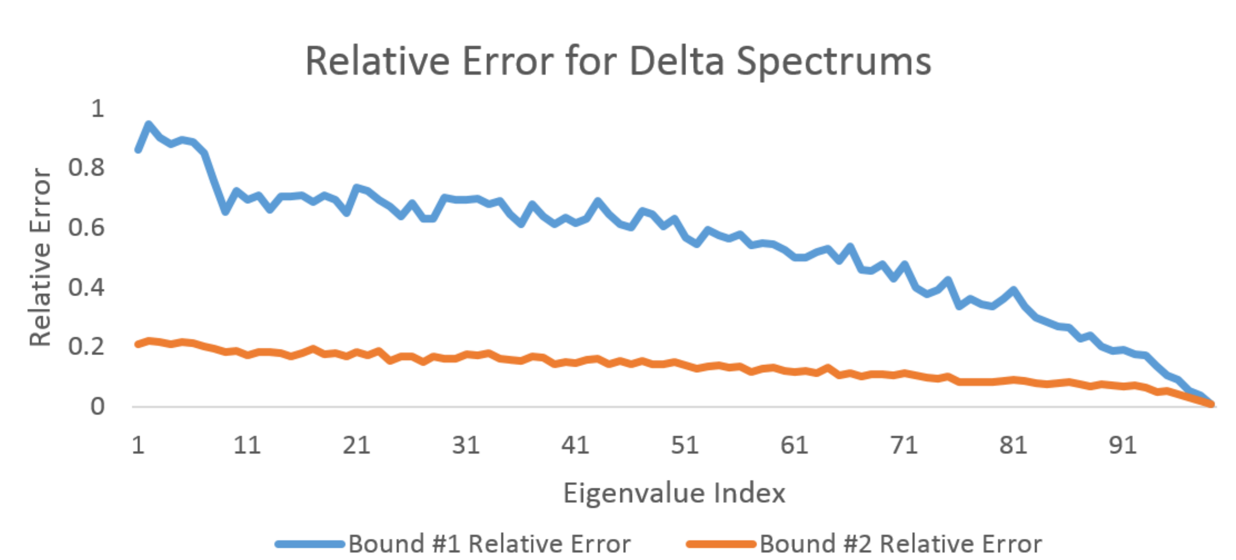
Let T be the diffusion matrix, k be the number of diffusion timesteps, and y be the initial graph signal. Then $x = T^k y$, and k_{min} is the value of k required for x to become (α,ω)-ABL. The following upperbounds on k_{min} were derived mathematically, based on two different assumptions:

Bound #1: $$k_{\min} \leq \frac{\log \frac{\alpha \hat{x}_1^2}{(1-\alpha)\|\hat{x}_{j+1:N}\|_2^2}}{2\log\left(\max_{i\in[j+1,N]}|\lambda_i|\right)}$$

Bound #2: $$k_{\min} \leq \max_{i\in[j+1,N]}\left(\frac{\log\frac{\alpha\hat{x}_1^2}{(1-\alpha)(N-j)\hat{x}_i^2}}{2\log|\lambda_i|}\right)$$

The performance of the two bounds was tested with code-generated graph signals:





Mathematically, the following are true for all ω:

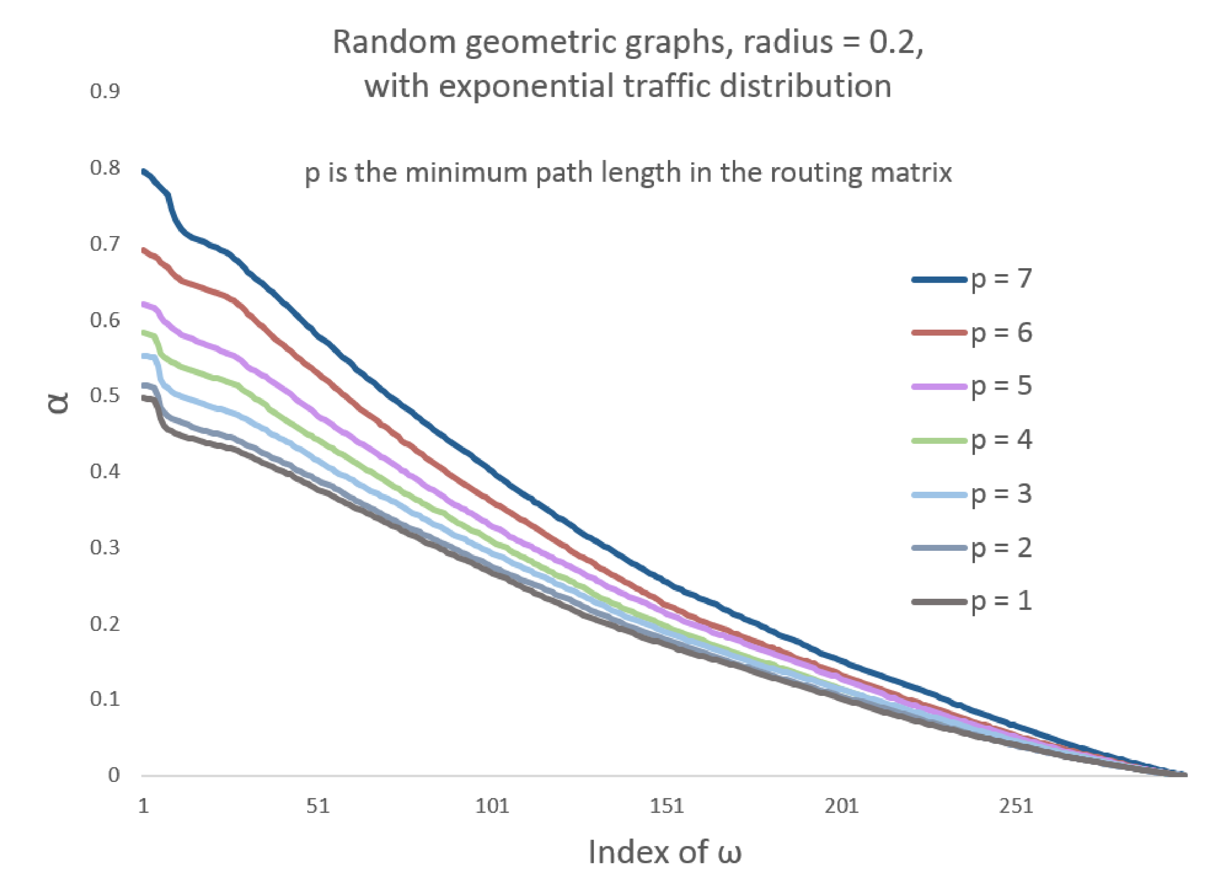
for non-bipartite graphs: $k \to \infty \implies \alpha = 0$

for bipartite graphs: $k \to \infty \implies \alpha = 1 - \dfrac{\hat{x}_1^2}{\hat{x}_1^2 + \hat{x}_N^2}$

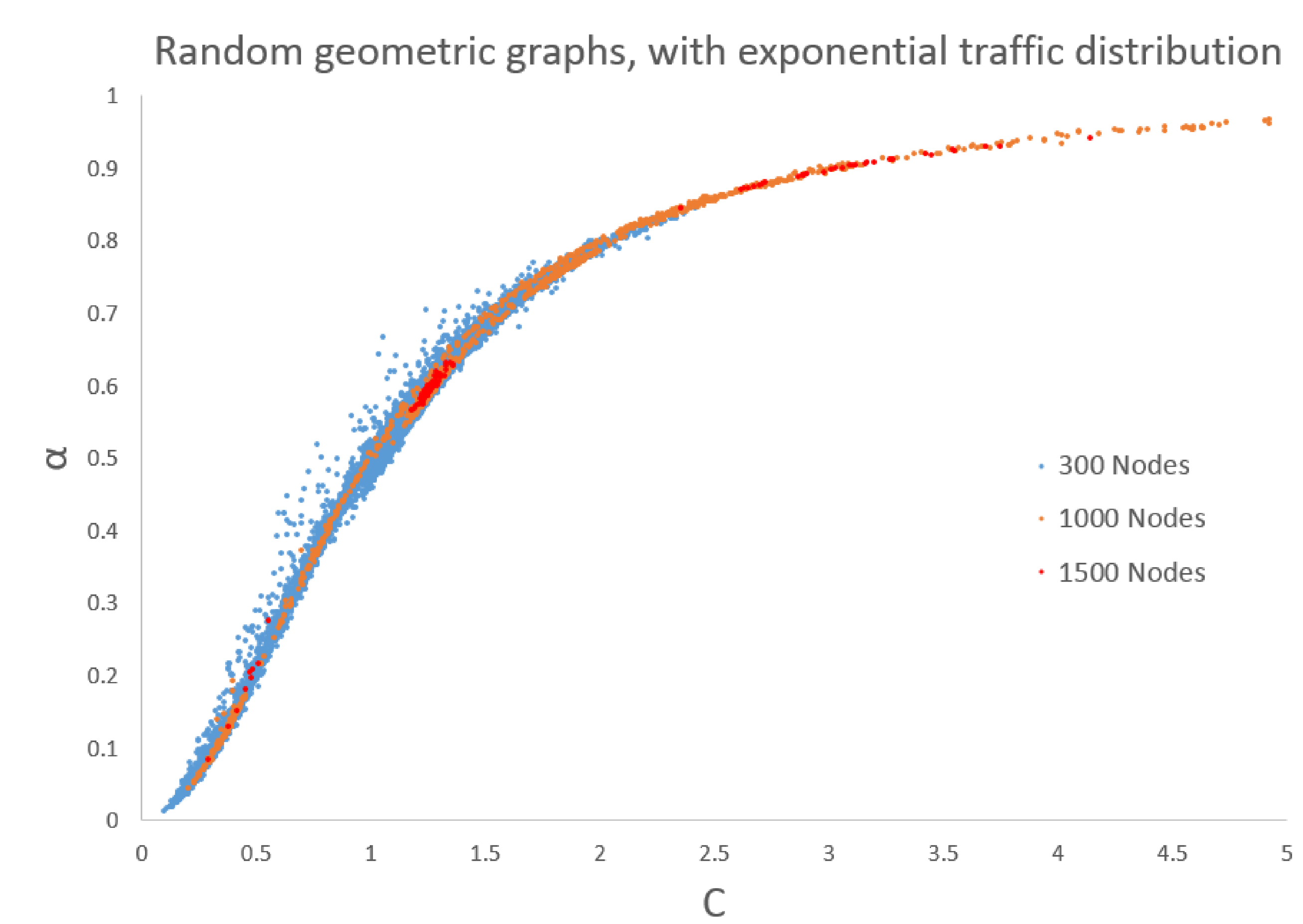
Estimating α in Traffic Models

Under what conditions is a traffic model (α,ω)-ABL? A traffic model can be represented by x = Ry, where R is the routing matrix, and y is the traffic intensity vector.

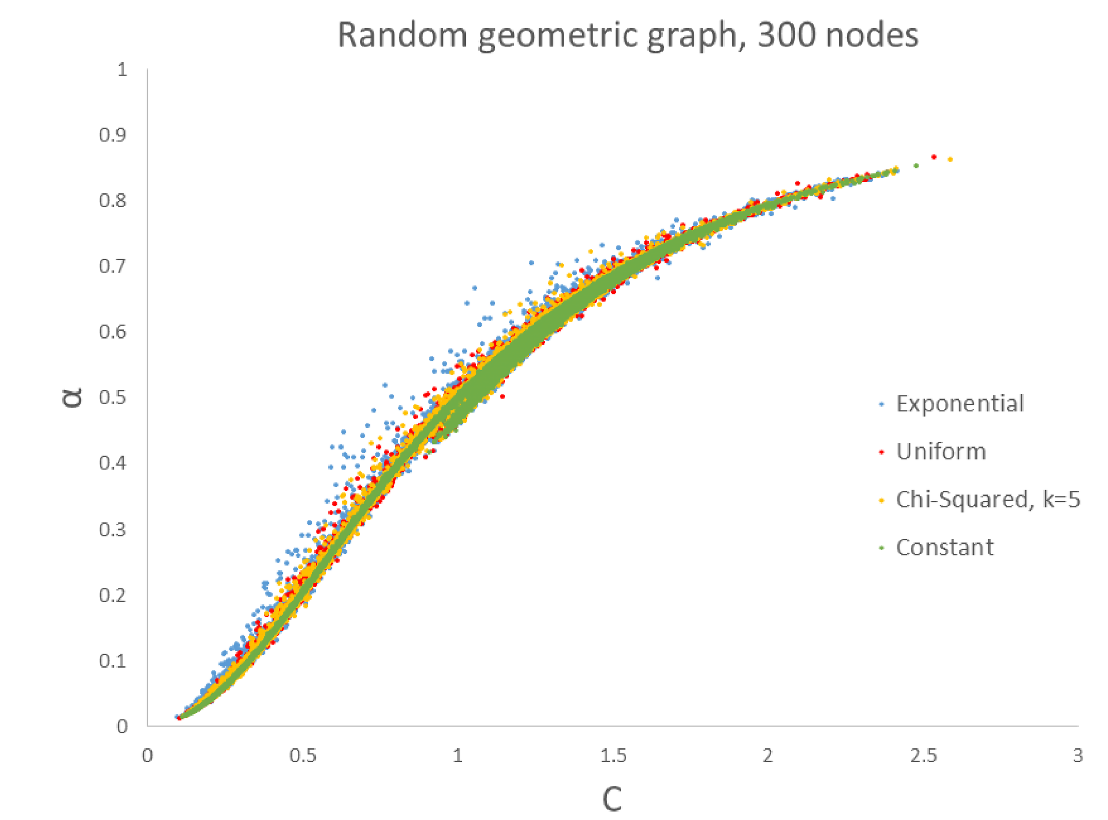
The graph below shows how α changed with ω, in a few of our randomly generated traffic models.



Let ψ(i) be the number of paths with traffic passing through node i. For random geometric graphs and ω = 0, α and C (the coefficient of variation of ψ) appear to be related by a sigmoid that is independent of graph size, and node connection radius.



The sigmoid also seems to be independent of the traffic distribution used in the model.



Below is an estimate of the mean of ψ, which is needed to calculate C. The equation comes from regression analysis on our dataset.

$$\mathrm{E}[\psi] \approx \varepsilon N(0.71 + \frac{0.285}{r})$$

ε = (# of paths in R)/(N choose 2)
N = number of nodes
r = node connection radius

The accuracy of the equation depends on how R is constructed, as shown below.

	Randomly select paths with no restriction on path length	Select all paths with length ≥ p
Mean Error	0.93%	35%
Maximum Error	6.16%	140%
# of Tests	22000	3200
Test Parameters	$N \in [300, 2000]$ $r \in [\sqrt{2\log N/N}, 1]$ $\varepsilon \in [0.01, 1]$	$N \in [300, 1200]$ $r \in [\sqrt{2\log N/N}, 1]$ $p \in [1, \text{max possible}]$

Future Work

- Find the value of α for ω > 0
- Determine an expression for $E[\psi^2]$

References

[1] H. Shomorony and A.S. Avestimehr. Sampling large data on graphs. In *Signal and Information Processing (GlobalSIP), 2014 IEEE Global Conference on,* pages 933-936, Dec 2014.

[2] Sunil K. Narang, Akshay Gadde, Eduard Sanon, and Antonio Ortega. Localized iterative methods for interpolation in graph structured data. CoRR, abs/1310.2646, 2013.